



**INDIAN SCHOOL SOHAR**  
**PRE-BOARD –II EXAMINATION (2023-24)**  
**MATHEMATICS (CODE -041)**

No. of pages: 5 + 1 graph

**SET - I**

**CLASS: XII**  
**DATE: 13/01/24**

**MAX. MARKS: 80**  
**TIME: 3 Hours**

**General Instructions:**

1. This question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) type questions of 2 marks each.
4. Section C has 6 Short Answer (SA) type questions of 3 marks each.
5. Section D has 4 Long Answer (LA) type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

<b>SECTION – A</b>		
<b>Multiple Choice Questions</b>		
<b>( Each question carries 1 mark)</b>		
1.	If $A = [a_{ij}]$ is a skew-symmetric matrix of order $n$ , then (a) $a_{ij} = \frac{1}{a_{ji}} \forall i, j$ (b) $a_{ij} \neq 0 \forall i, j$ (c) $a_{ij} = 0$ , where $i = j$ (d) $a_{ij} \neq 0$ where $i = j$	MARKS 1
2.	If $A$ is a square matrix of order 3 and $ A  = 5$ , then $ adjA $ is equal to a) 5                      b) 25                      c) 125                      d) $\frac{1}{5}$	1
3.	The function $f: R \rightarrow R$ given by $f(x) = - x - 1 $ is a) continuous as well as differentiable at $x = 1$ b) not continuous but differentiable at $x = 1$ c) continuous but not differentiable at $x = 1$ d) neither continuous nor differentiable at $x = 1$	1
4.	In the interval $(1, 2)$ the function $f(x) = 2 x - 1  + 3 x - 2 $ is a) Strictly Increasing                      b) Strictly Decreasing c) Neither Increasing nor Decreasing                      d) Remains constant	1
5.	If the points $A(3, -2)$ , $B(k, 2)$ and $C(8, 8)$ are collinear, then the value of $k$ is: a) 2                      b) -3                      c) 5                      d) 4	1
6.	If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ then $A^{-1}$ is a) $\begin{bmatrix} 2 & -3 \\ 5 & 2 \end{bmatrix}$ b) $\begin{bmatrix} -2 & 4 \\ 3 & 5 \end{bmatrix}$ c) $\frac{1}{19} \begin{bmatrix} 3 & -2 \\ 5 & 3 \end{bmatrix}$ d) $\frac{1}{19} A$	1
7.	Two dice are thrown. It is known that the sum of numbers on the dice is less than 6, the probability of getting a sum 3 is a) $1/18$ b) $2/5$ c) $1/5$ d) $5/18$	1
8.	The rate of change of the area of a circle with respect to its radius $r$ , at $r = 6$ cm is a) $10\pi$ b) $12\pi$ c) $8\pi$ d) $11\pi$	1

9	Evaluate : $\int_0^{\pi/2} \cos x e^{\sin x} dx$ a) 0    b) e    c) $e^{-1}$ d) $e^{\frac{\pi}{2}}$	1
10.	The vertices of the feasible region determined by some linear constraints are (0, 2), (1, 1), (3, 3), (1, 5). Let $Z = px + qy$ where $p, q > 0$ . The condition on $p$ and $q$ so that the maximum of $Z$ occurs at both the points (3, 3) and (1, 5) is a) $p = q$ b) $p = 2q$ c) $q = 2p$ d) $p = 3q$	1
11.	The direction ratios of the line which is perpendicular to the lines $\frac{x-7}{2} = \frac{y+17}{-3} = \frac{z-6}{1}$ and $\frac{x+5}{1} = \frac{y}{2} = \frac{z-4}{-2}$ is a) (4, 5, 7)    b) (4, -5, 7)    c) (4, -5, -7)    d) (-4, 5, 7)	1
12.	The region represented by the inequation $x - y \leq -1, x - y \geq 0, x \geq 0, y \geq 0$ is a) bounded    b) unbounded    c) does not exist    d) triangular region	1
13.	The area of a parallelogram whose adjacent sides represented by the vectors $2\hat{i} - 3\hat{k}$ and $4\hat{i} + 2\hat{j}$ is a) 10    b) 14    c) $\sqrt{11}$ d) $4\sqrt{14}$	1
14.	If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors of equal magnitude and angle between each pair of vectors is $\frac{\pi}{3}$ such that $ \vec{a} + \vec{b} + \vec{c}  = \sqrt{6}$ then $ \vec{a} $ is a) 2    b) 1    c) 1    d) $\sqrt{6}$	1
15.	If $\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ , then the projection of $\vec{a}$ on $\vec{b}$ is a) $1/7$ b) $5/7$ c) $8/7$ d) $9/7$	1
16.	The order and degree of the following differential equation are: $\left(1 + 3\frac{dy}{dx}\right)^{2/3} = 4\frac{d^3y}{dx^3}$ a) (1, 2/3)    b) (3, 1)    c) (3, 3)    d) (1, 2)	1
17.	Integrating factors of the differential equation $\cos x \frac{dy}{dx} + y \sin x = 1$ , is a) $\sin x$ b) $\sec x$ c) $\tan x$ d) $\cos x$	1
18.	$V$ is a matrix of order 3 such that $ \text{adj } V  = 7$ . Which of these could be $ V $ ? a) $7^2$ b) 7    c) $\sqrt{7}$ d) $\sqrt[3]{7}$	1
<b>ASSERTION-REASON BASED QUESTIONS</b> In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both (A) and (R) are true and (R) is the correct explanation of (A). (b) Both (A) and (R) are true but (R) is not the correct explanation of (A). (c) (A) is true but (R) is false. (d) (A) is false but (R) is true.		
19.	<b>Assertion (A):</b> Consider the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^3$ . Then $f$ is one-one <b>Reason (R):</b> Every polynomial function is one-one.	1

20.	<p><b>Assertion(A):</b> The vector equation of the line passing through the points (0, 5,2) and (1,4,5) is <math>\vec{r} = (5\hat{j} + 2\hat{k}) + \mu(\hat{i} - \hat{j} + 3\hat{k})</math></p> <p><b>Reason(R):</b> The vector equation of the line passing through the points <math>\vec{a}</math> and <math>\vec{b}</math> is <math>\vec{r} = \vec{a} + \mu(\vec{b} - \vec{a})</math></p>	1
<b>SECTION B</b> <b>(This section comprises of very short answer type questions (VSA) of 2 marks each)</b>		
21.	<p>Find the value of <math>\sin^{-1} \left[ \sin \left( \frac{13\pi}{7} \right) \right]</math></p> <p><b>OR</b></p> <p>Express <math>\tan^{-1} \left( \frac{\cos x}{1 - \sin x} \right)</math>, <math>-\frac{3\pi}{2} &lt; x &lt; \frac{\pi}{2}</math> in the simplest form.</p>	2
22.	<p>The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3cm/sec. How fast is the area decreasing when the two equal sides are equal to the base?</p> <p><b>OR</b></p> <p>The volume of the cube increases at a constant rate. Prove that the increase in its surface area varies inversely as the length of the side.</p>	2
23.	<p>The population of rabbits in a forest is modelled by the function below:  <math>P(t) = \frac{4000}{1 + e^{-0.5t}}</math> where P represents the population of rabbits in t years.  Determine whether the rabbit population is increasing or not, and justify your answer.</p>	2
24.	<p>Find the interval on which the function <math>f(x) = 2x^3 + 9x^2 + 12x - 1</math> is decreasing .</p>	2
25.	<p>Evaluate: <math>\int_0^{\frac{\pi}{2}} \frac{x + \sin x}{1 + \cos x} dx</math></p>	2
<b>SECTION C</b> <b>(This section comprises of short answer type questions (SA) of 3 marks each)</b>		
26.	<p>If <math>x = a(\theta - \sin\theta)</math>, <math>y = a(1 + \cos\theta)</math> find <math>\frac{d^2y}{dx^2}</math> at <math>\theta = \frac{\pi}{2}</math></p>	3
27.	<p>Find <math>\int \frac{\cos x}{(1 - \sin x)(2 - \sin x)} dx</math></p> <p><b>OR</b></p> <p>Find <math>\int \frac{\sin \phi}{\sqrt{\sin^2 \phi + 2 \cos \phi + 3}} d\phi</math></p>	3
28.	<p>Evaluate:  <math>\int_0^{\frac{\pi}{2}} (2 \log \sin x - \log \sin 2x) dx</math></p>	3
29.	<p>Solve the differential equation: <math>ye^{\frac{x}{y}} dx = \left( xe^{\frac{x}{y}} + y^2 \right) dy</math>, (<math>y \neq 0</math>).</p> <p><b>OR</b></p> <p>Solve the differential equation: <math>(\cos^2 x) \frac{dy}{dx} + y = \tan x</math>; <math>\left( 0 \leq x &lt; \frac{\pi}{2} \right)</math>.</p>	3

30.	<p>A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag?</p> <p><b>OR</b></p> <p>Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability distribution of the number of aces.</p>	3
31.	<p>Solve the following linear programming problem graphically.</p> <p>Maximize <math>Z = x + 2y</math></p> <p>Subject to constraints;</p> $x + 2y \geq 100$ $2x - y \leq 0$ $2x + y \leq 200$ $x, y \geq 0$	3
<p><b>SECTION D</b></p> <p><b>(This section comprises of long answer type questions (LA) of 5 marks each)</b></p>		
32.	<p>Show that <math>f: N \rightarrow N</math>, given by <math>f(x) = \begin{cases} x + 1, &amp; \text{if } x \text{ is odd} \\ x - 1, &amp; \text{if } x \text{ is even} \end{cases}</math> is a bijection.</p> <p><b>OR</b></p> <p>Let <math>N</math> denote the set of all natural numbers and <math>R</math> be the relation on <math>N \times N</math> defined by : <math>(a, b) R(c, d)</math> is <math>ad(b + c) = bc(a + d)</math>. Show that <math>R</math> is an equivalence relation</p>	5
33.	<p>Draw a rough sketch of the curve <math>y = 1 +  x + 1 </math>, <math>x = -3</math>, <math>x = 3</math>, <math>y = 0</math> and find the area of the region bounded by them using integration</p>	5
34.	<p>Solve the following system of equations by matrix method, where <math>x \neq 0, y \neq 0,</math></p> $z \neq 0$ $\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$ $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$ $\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$	5
35.	<p>Given below are two lines <math>L_1</math> and <math>L_2</math>:</p> <p><math>L_1: 2x = 3y = -z</math></p> <p><math>L_2: 6x = -y = -4z</math></p> <p>i) Find the angle between the two lines.</p> <p>ii) Find the shortest distance between the two lines.</p> <p><b>OR</b></p> <p>Find the length and the foot of the perpendicular drawn from the point <math>(2, -1, 5)</math> on the line</p> $\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \mu(10\hat{i} - 4\hat{j} - 11\hat{k})$ where $\mu$ is a scalar.	5
<p><b>SECTION E</b></p> <p><b>[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.]</b></p>		
36.	<p>The use of electric vehicles will curb air pollution in the long run. The use of electric vehicles is increasing every year and estimated number of electric vehicles in use at</p>	

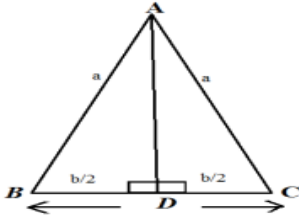
	any time $t$ is given by the function $V(t) = t^3 - 3t^2 + 3t - 100$ Where $t$ represents time and $t = 1, 2, 3, \dots$ corresponds to year 2021, 2022, 2023 $\dots$ respectively. Based on the above information answer the following:	
	(i) Can the above function be used to estimate number of vehicles in the year 2020? Justify.	1
	(ii) Prove that the function $V(t)$ is an increasing function	1
	(iii) Find the estimated number of vehicles in the year 2040.	2
	<b>OR</b>	
	(iii) Find the estimated number of vehicles in the year 2050.	
37.	Fighter jets are flying in a formation for an aero show as shown in the figure. Taking their control tower as the reference point and the reference point being origin, the coordinates of two fighters in their flight path are $A(10.5 \text{ km}, 10 \text{ km}, 1 \text{ km})$ and $B(10 \text{ km}, 10.5 \text{ km}, 0.9 \text{ km})$ . They are moving along the straight-line joining A and B at that point as seen in the figure.	
	i) What is the vector equation of the line passing through A and B?	1
	ii) What is the cartesian equation of the line passing through A and B?	1
	iii) What are the direction cosines of the line $\overrightarrow{AB}$ ?	2
	<b>OR</b>	
	iii) What is the angle made by the line $\overrightarrow{AB}$ with the positive direction of z-axis?	
38.	A shopkeeper sells three types of flower seeds A1, A2 and A3. They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of three types of seeds are 45%,60%,35%. Based on the given information, answer the following questions:	
	i) Find the probability of a randomly chosen seed to germinate	2
	ii) Find the probability that seed will not germinate given that it is of the type A3?	2

-----THE END -----

**MATHS PB-II-2023-24**  
**SCORING KEY-SET 1**  
**STD XII**

**(SECTION –A)**

<b>(SECTION –A)</b>		
1.	c) In a skew-symmetric matrix, the (i, j)th element is negative of the (j, i)th element. Hence, the (i, i)th element = 0	<b>MAR KS 1</b>
2.	b)25	1
3.	c)	1
4.	<i>If</i> $1 < x < 2$ then $f(x) = 2(x-1) - 3(x-2) = -x + 4$ $\therefore f'(x) = -1$ Hence $f(x)$ is Strictly decreasing function <u><b>Correct Answer is Option (b) Strictly Decreasing</b></u>	1
5.	c)5	1
6.	d) 1/19 A	1
7.	c)1/5	1
8.	b)12 pi	1
9	c)e <sup>-1</sup>	1
10	a) p=q	1
11	a) (4, 5, 7)	1
12	b) does not exist	1
13	b)14	1
14	c) 1	1
15	b) 5 /7	1
16	c) (3,3)	1
17	b) sec x	1
18	c) $\sqrt{7}$	1
19	C)	1
20	a)	1
<b>SECTIONB</b>		
21	$\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right] = \sin^{-1}\left[\sin\left(2\pi - \frac{\pi}{7}\right)\right]$ $= \sin^{-1}\left[\sin\left(-\frac{\pi}{7}\right)\right] = -\frac{\pi}{7}$ <p>OR</p> $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right) = \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2} - x\right)}{1 - \cos\left(\frac{\pi}{2} - x\right)}\right]$ $\tan^{-1}\left[\frac{2 \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) \cos\left(\frac{\pi}{4} - \frac{x}{2}\right)}{2 \sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}\right]$	1  1

	$\tan^{-1} \left[ \cot \left( \frac{\pi}{4} - \frac{x}{2} \right) \right] = \tan^{-1} \left[ \tan \frac{\pi}{2} - \left( \frac{\pi}{4} - \frac{x}{2} \right) \right]$ $\tan^{-1} \left[ \tan \left( \frac{\pi}{4} + \frac{x}{2} \right) \right] = \frac{\pi}{4} + \frac{x}{2}$	
22	<p>In isosceles <math>\Delta ABC</math>, let <math>AB = AC = a</math> and <math>BC = b</math> (given),</p> $\frac{da}{dt} = -3 \text{ cm/sec.}$ $AD = \sqrt{a^2 - \frac{b^2}{4}}$  <p>A = Area of <math>\Delta ABC = \frac{1}{2} (BC) (AD) = \left( \frac{1}{2} \right) (b) \sqrt{a^2 - \frac{b^2}{4}}</math></p> $\Rightarrow \frac{dA}{dt} = \left( \frac{b}{2} \right) \left( \frac{1}{2} \right) \left( a^2 - \frac{b^2}{4} \right)^{-1/2} \cdot \left( 2a \frac{da}{dt} \right)$ $= \frac{b(2a)(-3)}{4\sqrt{a^2 - \frac{b^2}{4}}} = \frac{-3ab}{2\sqrt{a^2 - \frac{b^2}{4}}}$ $\Rightarrow \left( \frac{dA}{dt} \right)_{a=b} = \frac{-3b^2}{2\sqrt{b^2 - \frac{b^2}{4}}} = \frac{-3b^2}{2\sqrt{\frac{3b^2}{4}}} = -\sqrt{3}b$ <p><math>\therefore</math> Area is decreasing at the rate of <math>\sqrt{3}b</math> cm/sec.</p> <p>OR</p> <p>Let the side of a cube be <math>x</math> unit.</p> <p>Volume of cube = <math>V = x^3</math></p> $\frac{dV}{dt} = 3x^2 \frac{dx}{dt} = k \text{ (constant)}$ $\frac{dx}{dt} = \frac{k}{3x^2}$ <p>Surface area = <math>S = 6x^2</math></p> $\frac{dS}{dt} = 12x \frac{dx}{dt}$ $\frac{dS}{dt} = 12x \frac{k}{3x^2} = 4 \left( \frac{k}{x} \right)$ <p>Hence, the surface area of the cube varies inversely as length of side</p>	1  1
23	$P'(t) = 2000 \times \frac{(-1)}{(1+e^{-0.5t})^2} \times e^{-0.5t} \times \left( -\frac{1}{2} \right) = \frac{2000(e^{-0.5t})}{(1+e^{-0.5t})^2}$ <p>Writes that the above quantity is greater than 0 for any value of <math>t</math>.</p> <p>Concludes that the rabbit population is increasing.</p>	1  1

24	$f(x) = 2x^3 + 9x^2 + 12x - 1$ $f'(x) = 6x^2 + 18x + 12$ $= 6(x+2)(x+1)$ $x = -2, -1$ $f(x) \text{ is decreasing in } [-2, -1].$	1  1
25	$I = \int_0^{\frac{\pi}{2}} \frac{x+2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} dx$ $= \int_0^{\frac{\pi}{2}} \frac{x}{2} \sec^2 \frac{x}{2} dx + \int_0^{\frac{\pi}{2}} \tan \frac{x}{2} dx$ <p>Using by parts for 1 st integral</p> $= \left[ x \tan \frac{x}{2} \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \tan \frac{x}{2} dx + \int_0^{\frac{\pi}{2}} \tan \frac{x}{2} dx = \frac{\pi}{2}$	1+1
<b>SECTION C</b>		
26	<p>Sol. <math>x = a(\theta - \sin \theta) \Rightarrow \frac{dx}{d\theta} = a(1 - \cos \theta)</math></p> <p><math>y = a(1 + \cos \theta) \Rightarrow \frac{dy}{d\theta} = a(-\sin \theta)</math></p> $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a(-\sin \theta)}{a(1 - \cos \theta)} = -\frac{2 \sin \theta / 2 \cdot \cos \theta s \theta}{2 \sin^2 \theta / 2} = -\cot \frac{\theta}{2}$ $\frac{d^2y}{dx^2} = \operatorname{cosec}^2 \frac{\theta}{2} \cdot \frac{1}{2} \cdot \frac{d\theta}{dx} = \frac{1}{2} \operatorname{cosec}^2 \frac{\theta}{2} \cdot \frac{1}{a(1 - \cos \theta)}$ $\left( \frac{d^2y}{dx^2} \right)_{\theta = \frac{\pi}{2}} = \frac{1}{2} \operatorname{cosec}^2 \frac{\pi}{4} \cdot \frac{1}{a \left( 1 - \cos \frac{\pi}{2} \right)} = \frac{1}{2} \cdot 2 \cdot \frac{1}{a} = \frac{1}{a}$	1+1+ 1
27	$I = \int \frac{\cos x}{(1 - \sin x)(2 - \sin x)} dx$ $= \int \frac{\cos x}{(\sin x - 1)(\sin x - 2)} dx$ $\Rightarrow I = \int \frac{dt}{(t-1)(t-2)}$ $= \int \left( \frac{-1}{t-1} + \frac{1}{t-2} \right) dt$ $= -\log  t-1  + \log  t-2 $ $= \log \left  \frac{t-2}{t-1} \right  + c$ $= \log \left  \frac{\sin x - 2}{\sin x - 1} \right  + c$ <p style="text-align: center;"><b>OR</b></p> $I = \int \frac{\sin \phi}{\sqrt{\sin^2 \phi + 2 \cos \phi + 3}} d\phi$ $= \int \frac{\sin \phi}{\sqrt{1 - \cos^2 \phi + 2 \cos \phi + 3}} d\phi$ $= \int \frac{\sin \phi d\phi}{\sqrt{4 + 2 \cos \phi - \cos^2 \phi}}$ <p style="text-align: right; margin-right: 100px;">put <math>\cos \phi = t</math> - <math>\sin \phi d\phi = dt</math></p> $= \int \frac{-dt}{\sqrt{4 + 2t - t^2}} = -\int \frac{dt}{\sqrt{-(t^2 - 2t - 4)}}$ $= -\int \frac{dt}{\sqrt{-(t^2 - 2t + 1 - 5)}} = -\int \frac{dt}{\sqrt{(\sqrt{5})^2 - (t-1)^2}}$ $= -\sin^{-1} \left( \frac{t-1}{\sqrt{5}} \right) + c = -\sin^{-1} \left( \frac{\cos \phi - 1}{\sqrt{5}} \right) + c$	1  1  1



28	$I = \int_0^{\frac{\pi}{2}} \log \left( \frac{\sin^2 x}{2 \sin x \cos x} \right) dx$ $I = \int_0^{\frac{\pi}{2}} \log \left( \frac{\tan x}{2} \right) dx$ $I = \int_0^{\frac{\pi}{2}} \log \left( \frac{\tan \left( \frac{\pi}{2} - x \right)}{2} \right) dx = I = \int_0^{\frac{\pi}{2}} \log \left( \frac{\cot x}{2} \right) dx$ $2I = \int_0^{\frac{\pi}{2}} \log \left( \frac{\tan x}{2} \right) \left( \frac{\cot x}{2} \right) dx = \int_0^{\frac{\pi}{2}} \log \left( \frac{1}{4} \right) dx$ $I = \frac{\pi}{4} \log \left( \frac{\pi}{4} \right)$	1  1  1
29	<p><b>Method 2:</b> We have, <math>\frac{dx}{dy} = \frac{xe^{\frac{x}{y}} + y^2}{y \cdot e^{\frac{x}{y}}}</math></p> $\Rightarrow \frac{dx}{dy} = \frac{x}{y} + \frac{y}{e^{\frac{x}{y}}} \dots \dots \dots (i)$ <p>Let <math>x = vy \Rightarrow \frac{dx}{dy} = v + y \cdot \frac{dv}{dy}</math> ;</p> <p>So equation (i) becomes <math>v + y \frac{dv}{dy} = v + \frac{y}{e^v}</math></p> $\Rightarrow y \frac{dv}{dy} = \frac{y}{e^v}$ $\Rightarrow e^v dv = \frac{y}{dy}$ <p>On integrating we get, <math>\int e^v dv = \int \frac{y}{dy} \Rightarrow e^v = y + c \Rightarrow e^{x/y} = y + c</math></p> <p>OR</p> $(\cos^2 x) \frac{dy}{dx} + y = \tan x$ <p>Dividing both the sides by <math>\cos^2 x</math>, we get</p> $\frac{dy}{dx} + \frac{y}{\cos^2 x} = \frac{\tan x}{\cos^2 x}$ $\frac{dy}{dx} + y(\sec^2 x) = \tan x(\sec^2 x) \dots \dots \dots (i)$ <p>Comparing with <math>\frac{dy}{dx} + Py = Q</math></p> $P = \sec^2 x, Q = \tan x \cdot \sec^2 x$ <p>The Integrating factor is, <math>IF = e^{\int P dx} = e^{\int \sec^2 x dx} = e^{\tan x}</math></p> <p>On multiplying the equation (i) by <math>e^{\tan x}</math>, we get</p> $\frac{d}{dx} (y \cdot e^{\tan x}) = e^{\tan x} \tan x (\sec^2 x) \Rightarrow d(y \cdot e^{\tan x}) = e^{\tan x} \tan x (\sec^2 x) dx$ <p>On integrating we get, <math>y \cdot e^{\tan x} = \int t \cdot e^t dt + c_1</math>; where, <math>t = \tan x</math> so that <math>dt = \sec^2 x dx</math></p> $= te^t - e^t + c = (\tan x) e^{\tan x} - e^{\tan x} + c$ <p><math>\therefore y = \tan x - 1 + c \cdot (e^{-\tan x})</math>, where '<math>c_1</math>' &amp; '<math>c</math>' are arbitrary constants of integration.</p>	1  1  1

30	2/3 OR			1 1 1	
		X	0	1	2
		P(X)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

31

Corner Points	$Z = 3x + y$
A (0, 50)	50
B (0, 200)	200
C (50, 100)	250
D (20, 40)	100

**Max  $z = 250$  at  $x = 50, y = 100$**

**SECTION D**

32

Prove by taking different cases  
OR

We have :  $(a, b) R (c, d) \Rightarrow ad (b + c) = bc (a + d)$  on  $N$ .

(i)  $(a, b) R (a, b) \Rightarrow ab(b + a) = ba (a + b) \Rightarrow ab(a + b) = ab (a + b)$ , which is true. Thus R is reflexive.

(ii)  $(a, b) R (c, d) \Rightarrow ad(b + c) = bc (a + d) \Rightarrow bc(a + d) = ad (b + c) \Rightarrow cb(d + a) = da (c + b)$  [ $\because bc = cb$  and  $a + d = d + a$ ; etc.  $\forall a, b, c, d \in N$ ]  $\Rightarrow (cb) R (a, b)$ . Thus R is symmetric

(iii) Let  $(a, b) R (c, d)$  and  $(c, d) R (e, f) \therefore ad (b + c) = bc (a + d)$  and  $cf(d + e) = de(c + f) \Rightarrow be(a+f) = af(b+e) \Rightarrow af(b+e) = be(a+f) \Rightarrow (a, b) R (e, f)$ . Thus R is transitive.

Hence, R is an equivalence relation.

33

Required Area =  $\int_{-3}^{-1} -x dx + \int_{-1}^3 (x+2) dx$

$$= \left[ -\frac{x^2}{2} \right]_{-3}^{-1} + \left[ \frac{x^2}{2} + 2x \right]_{-1}^3$$

$$= -\frac{1}{2}(1-9) + \left[ \left( \frac{9}{2} + 6 \right) - \left( \frac{1}{2} - 2 \right) \right]$$

$$= 4 + 12 = 16 \text{ square units}$$

34	$ A  = \begin{vmatrix} 2 & -3 & 3 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{vmatrix}$ $= 2(2+1) + 3(2-3) + 3(-1-3)$ <p style="text-align: center;">[expanding along <math>R_1</math>]</p> $= 6 - 3 - 12 = -9 \neq 0$ $\begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \frac{1}{-9} \begin{bmatrix} 3 & 3 & -6 \\ 1 & -5 & 1 \\ -4 & -7 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$ $= \frac{-1}{9} \begin{bmatrix} 30 + 30 - 78 \\ 10 - 50 + 13 \\ -40 - 70 + 65 \end{bmatrix}$ $= \frac{-1}{9} \begin{bmatrix} -18 \\ -27 \\ -45 \end{bmatrix}$ <p><math>X = 1/2, y = 1/3, z = 1/5</math></p>	1  1  1  1  1
35	<p>Rewrites the equation of <math>L_1</math> in cartesian form as:</p> $\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ <p>Rewrites the equation of <math>L_2</math> in cartesian form as:</p> $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$ <p>i) Identifies the direction cosines of both the lines as <math>(3, 2, -6)</math> and <math>(2, -12, -3)</math>.</p> <p>Finds the cosine of the angle between the two lines as:</p> $\cos \theta = \left  \frac{6-24+18}{\sqrt{49}\sqrt{157}} \right  = 0$ <p>(Award 0.5 marks if only the formula of the cosine of the angle between the two lines is written correctly.) Concludes that the angle between the two lines is <math>90^\circ</math>.</p> <hr/> <p>ii) Rewrites the equations of <math>L_1</math> and <math>L_2</math> in vector form as:</p> $\vec{r}_1 = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 6\hat{k}), \text{ where } \lambda \in \mathbb{R}$ $\vec{r}_2 = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda(2\hat{i} - 12\hat{j} - 3\hat{k}), \text{ where } \lambda \in \mathbb{R}$ <p>Writes that both the lines pass through the origin hence intersect at the origin.</p> <p>(Award full marks if the inference about both lines passing through the origin is drawn without writing the vector forms.)</p> <p>Writes that since both the lines intersect at the origin, the shortest distance between the two lines is 0 units.</p> <p>OR Constant <math>k = -1</math> Foot of perpendicular <math>= (1, 2, 3)</math> Length <math>= \sqrt{14}</math></p>	1  1  1  1  1  1

**SECTION E**

<p style="text-align: center;"><math>V(t) = t^3 - 3t^2 + 3t - 100</math></p> <p>(i) No, the above function cannot be used to estimate number of vehicles in the year 2020 because for 2020 we have <math>t = 0</math> and  <math>V(0) = 0 - 0 + 0 - 100 = -100</math>  Which is not possible</p> <p style="text-align: center;"><math>V'(t) = 3t^2 - 6t + 3</math>  <math>= 3(t^2 - 2t + 1)</math>  <math>= 3(t - 1)^2 \geq 0.</math></p> <p>Hence <math>V(t)</math> is always increasing function.</p> <p>ii) <math>V(20) = (20)^3 - 3(20)^2 + 3(20) - 100</math>  therefore, the estimated number of vehicles in the year 2040 are 6760.</p> <p style="text-align: center;"><math>V(30) = \overset{DR}{=}</math></p>	1 1 2
<p>37i) <math>\vec{r} = 10.5\hat{i} + 10\hat{j} + \hat{k} + \lambda(-0.5\hat{i} + 0.5\hat{j} - 0.1\hat{k})</math></p> <p>ii) <math>\frac{x - 10.5}{-0.5} = \frac{y - 10}{0.5} = \frac{z - 1}{-0.1}</math></p> <p style="text-align: center;"><math>\frac{-0.5}{\sqrt{0.5^2 + 0.5^2 + 0.1^2}}, \frac{0.5}{\sqrt{0.5^2 + 0.5^2 + 0.1^2}}, \frac{-0.1}{\sqrt{0.5^2 + 0.5^2 + 0.1^2}}</math></p> <p style="text-align: center;"><math>\frac{-0.5}{\sqrt{0.51}}, \frac{0.5}{\sqrt{0.51}}, \frac{-0.1}{\sqrt{0.51}}</math></p> <p>iii) OR  <math>\emptyset = \frac{-0.1}{\sqrt{0.51}}</math></p>	1 1 2
<p>38. (i) 0.49 (ii) 0.65</p>	2 2

MATHS PRE BOARD II -2023-24

STD XII

1. This question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) type questions of 2 marks each.
4. Section C has 6 Short Answer (SA) type questions of 3 marks each.
5. Section D has 4 Long Answer (LA) type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

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Sl.no	CHAPTER	MCQ1 MARK	CBQ 4 MARK	2 MARKS	3 MARKS	5 MARKS	Total 38(80 marks)
1	<b>Relation and Function</b>	<b>1 (AR)</b>				<b>1*</b>	<b>2 (6 marks)</b>

2	<b>Inverse Trigonometry</b>			<b>1*</b>			<b>1(2 marks)</b>
3	<b>Matrices</b>	<b>1</b>					<b>1(1marks)</b>
4	<b>Determinants</b>	<b>1+1+1+1</b>				<b>1</b>	<b>5 (9 marks)</b>
5	<b>Continuity and Differentiability</b>	<b>1</b>			<b>1</b>		<b>2( 4 marks)</b>
6	<b>Application of Derivatives</b>	<b>1 + 1(AR)</b>	<b>1</b>	<b>1+1+1*</b>			<b>6(12 marks)</b>
7	<b>Integrals</b>	<b>1</b>		<b>1</b>	<b>1+1*</b>		<b>4(9 marks)</b>
8	<b>Application of Integrals</b>					<b>1</b>	<b>1(5 marks)</b>
9	<b>Differential Equation</b>	<b>1+1</b>			<b>1*</b>		<b>3(5 marks)</b>
10	<b>Vectors</b>	<b>1+1+1</b>					<b>3( 3marks)</b>
11	<b>3 D</b>	<b>1+1</b>	<b>1</b>			<b>1*</b>	<b>4( 11 marks)</b>
12	<b>Probability</b>	<b>1</b>	<b>1</b>		<b>1</b>		<b>3( 8 marks)</b>
13	<b>Linear Programming</b>	<b>1+1</b>			<b>1*</b>		<b>3(5 marks)</b>

Each student 1 graph